Problem 3.48 p. 82

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Solution

a) We use the definition of the conditional probability:

\[ P(\text{Pacific} \cap \text{Soda}) = P(\text{Soda}|\text{Pacific}) \cdot P(\text{Pacific}) = 0.71 \cdot 0.04 = 0.0284 \]

b) We use the total probability formula:

\[
P(\text{Pop}) = P(\text{Pop}|\text{Pacific}) \cdot P(\text{Pacific})
\]

\[
+ P(\text{Pop}|\text{Rockies}) \cdot P(\text{Rockies}) + P(\text{Pop}|\text{Southwest}) \cdot P(\text{Southwest})
\]

\[
+ P(\text{Pop}|\text{Midwest}) \cdot P(\text{Midwest}) + P(\text{Pop}|\text{Northeast}) \cdot P(\text{Northeast})
\]

\[
+ P(\text{Pop}|\text{Southeast}) \cdot P(\text{Southeast})
\]

\[
= 0.15 \cdot 0.04 + 0.61 \cdot 0.10 + 0.12 \cdot 0.06 + 0.70 \cdot 0.18
\]

\[
+ 0.30 \cdot 0.28 + 0.18 \cdot 0.34 = 0.3454
\]

In R:

\[
> 0.15*0.04 + 0.61*0.10+0.12*0.06+0.70*0.18+0.30*0.28+0.18*0.34
\]

\[
[1] 0.3454
\]

> 

c) We need \( P(\text{Southeast}|\text{Coke}) \). This is an ideal set-up to use Bayesian Formula. For practical reasons, we split the calculation into two formulas. The next formula is essentially the Bayesian Formula, but we need to rewrite \( P(\text{Coke}) \) using total probability formula:

\[
P(\text{Southeast}|\text{Coke}) = \frac{P(\text{Coke}|\text{Southeast}) \cdot P(\text{Southeast})}{P(\text{Coke})}
\]

\[
P(\text{Coke}) = P(\text{Coke}|\text{Pacific}) \cdot P(\text{Pacific})
\]

\[
+ P(\text{Coke}|\text{Rockies}) \cdot P(\text{Rockies}) + P(\text{Coke}|\text{Southwest}) \cdot P(\text{Southwest})
\]

\[
+ P(\text{Coke}|\text{Midwest}) \cdot P(\text{Midwest}) + P(\text{Coke}|\text{Northeast}) \cdot P(\text{Northeast})
\]

\[
+ P(\text{Coke}|\text{Southeast}) \cdot P(\text{Southeast})
\]

\[
= 0.14 \cdot 0.04 + 0.08 \cdot 0.10 + 0.65 \cdot 0.06 + 0.04 \cdot 0.18
\]

\[
+ 0.02 \cdot 0.28 + 0.39 \cdot 0.34 = 0.198
\]

Also, \( P(\text{Coke}|\text{Southeast}) \cdot P(\text{Coke}) = 0.39 \cdot 0.34 = 0.1326 \) and finally:

\[
P(\text{Coke}) = 0.669697
\]

\[
> a <-0.14*0.04+0.08*0.10+0.65*0.06+0.04*0.18+0.02*0.28+0.39*0.34; a
\]

\[
[1] 0.198
\]

\[
> b <-0.39*0.34; b
\]

\[
[1] 0.1326
\]

\[
> b/a
\]

\[
[1] 0.669697
\]

>