The statement of exercise 4.17

You are to pay $1 to play a game that consists of drawing one ticket at random from a box of numbered tickets. You win the amount (in dollars) of the number on the ticket you draw. The following two boxes of numbered tickets are available.

I. 0,1,2

II. 0,0,0,1,4

a) Find the expected value and variance of your net gain per play with box I.

b) Repeat part (a) for box II.

c) Given that you have decided to play, which box would you choose and why.

Solution

a

We use R:
\begin{verbatim}
> p.I <- c(1/3, 1/3, 1/3); x.I <- c(0, 1, 2) - 1
> mu.I <- sum(x.I * p.I)
> var.I <- sum((x.I - mu.I)^2 * p.I)
> mu.I; var.I;

[1] 0
[1] 0.6666667

> p.II <- c(1/5, 1/5, 1/5, 1/5, 1/5); x.II <- c(0, 0, 0, 1, 4) - 1
> mu.II <- sum(x.II * p.II)
> var.II <- sum((x.II - mu.II)^2 * p.II);
> mu.II; var.II

[1] 5.551115e-17
\end{verbatim}
Setup graphics

The following commands are particular to $\TeX_{\text{MACS}}$ and fix the graphics parameters. It is not necessary to include this if you are not using $\TeX_{\text{MACS}}$.

```r
> opts = options(); opts\$\text{texmacs}\$width=7.5;
  opts\$\text{texmacs}\$height=7.5; opts\$\text{texmacs}\$nox11=F; options(opts)
> X11(pointsize=8, height=3, width=3)
```

Both boxes have the same expected value of 0 of the gain per game (zero-sum game), but the variance of the second box is significantly higher. Higher variance means that more often the real winnings are significantly higher than the expec-
tation. Hence, box II should be chosen for a typical gambling objective of maximizing the probability of the extreme value. Note that the probability of simply losing $1 is significantly higher for box II. The distribution of box I is symmetric about 0, i.e. winnings and losses are completely symmetric, which is another reason why box I is unattractive to a gambler.

We conduct a quick simulation to see the winnings as function of time, as we play many games in successions.

```r
> n<-1000
> w1 <- cumsum(sample(x.I,n,replace=T))
> w2 <- cumsum(sample(x.II,n,replace=T))
```

The following plot plots the difference between true winnings and expected winnings over time, for both choices.

```r
> matplot(1:n,cbind(w1-1:n,w2-1:n));v()
```
cbind(w1 - 1:n, w2 - 1:n)