Problem 4.2, p. 100

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The problem

Among 12 applicants for an open position, 7 are women and 5 are men. Suppose that three applicants are randomly selected from the applicant pool for final interviews. Let $X$ be the number of female applicants among the final three.

a) Find the probability function for $X$.

b) Graph the probability function of $X$.

c) Find the distribution function of $X$.

d) Graph the distribution function of $X$. 
Solution

a

The probability function is given by:

\[ p(x) = \binom{7}{x} \binom{5}{3-x} \binom{12}{3} \]

(Later on we learn that this is the hypergeometric distribution.)

The support is \{0, 1, 2, 3\}, i.e. \( p(x) > 0 \) only for \( x = 0, 1, 2, 3 \).

b

We use R:

\[
> x <- c(0,1,2,3)
\]
> p <- choose(7,x)*choose(5,3-x)/choose(12,3); p

[1] 0.04545455 0.31818182 0.47727273 0.15909091

> barplot(p); v()
The cumulative distribution function is:

\[ F(x) = \sum_{y \leq x} p(x) = \sum_{y \leq x} \frac{\binom{7}{x} \binom{5}{3-x}}{\binom{12}{3}} \]

There is no close form for it, but using R it is easy to find its values using `cumsum`.

```r
> q <- cumsum(p)
```
We produce the plot with R.

```r
> barplot(q);v()
```