1 The statement of exercise 5.11

The proportion of time during a 40-hour workweek that an industrial robot was in operation was measured for a large number of weeks. The measurement can be modeled by the probability density function

\[ f(x) = \begin{cases} 
2x, & 0 \leq x \leq 1 \\
0, & \text{otherwise.} 
\end{cases} \]

If \( X \) denotes the proportion of time this robot will be in operation during a coming week, find the following.

a) \( P(X > 1/2) \)

b) \( P(X > 1/2 | X > 1/4) \)

c) \( P(X > 1/4 | X > 1/2) \)

d) \( F(x) \). Graph the function. Is \( F(x) \) continuous?

2 Solution

2.1 Parts a-c

The various probabilities are integrals.

\[
\begin{align*}
P(X > 1/2) &= \int_{-\infty}^{\infty} f(x) \, dx = \int_{1/2}^{1} 2x \, dx = \left[ x^2 \right]_{1/2}^{1} = \frac{3}{4} \\
P(X > 1/4) &= \frac{15}{16}
\end{align*}
\]
\[ P(X > 1/2 | X > 1/4) = \frac{P(X > 1/2)}{P(X > 1/4)} = \frac{3/4}{15/16} = \frac{3 \cdot 16}{15 \cdot 4} = \frac{4}{5} = 0.8. \]
\[ P(X > 1/4 | X > 1/2) = 1 \quad \text{(note: } X > 1/2 \text{ implies } X > 1/4) \]

2.2 Part d

We find easily from the equation \( F'(x) = f(x) \) that

\[
F(x) = \begin{cases} 
0, & x < 0 \\
x^2, & 0 \leq x \leq 1 \\
1, & x > 1 
\end{cases}
\]

The function is continuous. Every function that is differentiable (everywhere!) is continuous. So, \( F(x) \) is always continuous.

We use R to plot the function.

\[ \text{Figure 1. The distribution of the proportion of the work week that the robot is in operation.} \]
3 The R code

```r
> ##
> ## File: robot.R
> ##
> ## Author: Marek Rychlik (rychlik@u.arizona.edu)
> ## Description: Code to solve exercise 5.11
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> ##

## Define the Jerry distribution for scalar arguments
F0 <- function(x) {
  if(x<0) 0
  else if(x<1) x^2
  else 1
}

## Convert F0 to a vector function
F <- Vectorize(F0)

## Plot
png(filename="robot_cdf.png")
plot(F,-1,2)
de.v.off()
```