1 The statement of exercise 5.8

Jerry is always early for appointments, arriving between 10 minutes early to exactly on time. The distribution function associated with $X$, the number of minutes early he arrives, is as follows:

$$F(x) = \begin{cases} 
0, & x < 0 \\
\frac{x^2}{40}, & 0 \leq x \leq 4 \\
\frac{20x - x^2 - 40}{60}, & 4 \leq x \leq 10 \\
1, & x > 10 
\end{cases}$$

a) Graph the distribution function.

b) Find the probability that Jerry arrives at least 5 minutes early.

c) Find the probability density function of $X$.

d) Graph the probability density function of $X$.

2 Solution

2.1 a

We use R. It may be instructional how it is done. First we define a function $F_0$ which only works correctly for scalar arguments. Then
we use Vectorize to convert it to \( F \) which behaves correctly for vector arguments. Then we simply call plot. As the amount of R code is large, we use an external script. The code is included at the end.

![Figure 1. Jerry’s distribution (c.d.f).](image)

2.2 b

This is \( P(X \geq 5) = 1 - P(X \leq 5) = 1 - F(5) \). We have

\[
F(5) = \frac{20 \cdot 5 - 5^2 - 40}{60} = \frac{100 - 25 - 40}{60} = \frac{35}{60} = \frac{7}{12} = 0.58333...
\]

Hence,

\[
1 - F(5) = \frac{5}{12} = 0.41666...
\]
2.3 c

We need to find the derivative of $F(x)$. Despite the fact that the function is defined piecewise, the pieces match so that the derivative exists and is continuous. Its shape can be described as “triangular”.

$$f(x) = \begin{cases} 
0, & x \leq 0 \\
\frac{2x}{20}, & 0 \leq x \leq 4 \\
\frac{20 - 2x}{60}, & 4 \leq x \leq 10 \\
0, & x > 10 
\end{cases}$$

2.4 d

![Figure 2](image)

**Figure 2.** Jerry’s probability density function.

3 The R code
```r
## File: jerry.R
## Author: Marek Rychlik (rychlik@u.arizona.edu)
## Description: Code to solve exercise 5.8
## Copyright: (C) Marek Rychlik, 2010, All rights reserved
##
## Define the Jerry distribution for scalar arguments
F0 <- function(x) {
  if(x<0) 0
  else if(x<4) x^2/40
  else if(x<10) (20*x-x^2-40)/60
  else 1
}

## Convert F0 to a vector function
F <- Vectorize(F0)

## Test F
F(1:4)

## Plot
png(filename="jerry_cdf.png")
plot(F,-1,11)
dev.off()

## Define the Jerry density and plot it
f0 <- function(x) {
  if(x<0) 0
  else if(x<4) 2*x/40
  else if(x<10) (20-2*x)/60
  else 0
}

f <- Vectorize(f0)
png(filename="jerry_pdf.png")
plot(f,-1,11)
dev.off();
```