MAREK RYCHLIK

Version of March 12, 2010

6770 N Table Mountain Rd
Tucson, AZ 85718
USA

E-mail: rychlik@u.arizona.edu
(520) 219-2837 (home)
(240) 855-2239 (cell)

Employment History

2007–2008	Chief Science Officer, QBit, LLC, later Qbit Corp., Gaithersburg, MD.
2005–2006	Senior Research Mathematician, QBit, LLC, Bethesda, MD.
2000–present	Professor of Mathematics, University of Arizona, Tucson.
1994–2000	Associate Professor of Mathematics with tenure, University of Arizona, Tucson.
1989–1994	Assistant Professor of Mathematics (tenure-track), University of Arizona, Tucson.
1987–1989	Visiting Member, Institute for Advanced Study, Princeton, New Jersey.
1984–1987	Acting Assistant Professor, University of Washington, Seattle.
1983–1984	Teaching Associate and Visiting Lecturer, University of California, Berkeley.
1983	Teaching Assistant, University of California, Berkeley.
1979–1982	Teaching and Research Assistant, Warsaw University, Poland.

Education and Degrees Earned

Ph.D. Mathematics University of California, Berkeley, 1983.

Master's Mathematics University of Warsaw, Poland, 1979.

Honors and Awards

The Monroe Martin Prize for an outstanding paper in applied mathematics for an author under the age of 35. The Monroe Martin Prize[1] is awarded by the Institute for Physical Science and Technology, University of Maryland, College Park, Maryland, 1991.

NSF grant awards 1985/86, 1986/87, 1990/1991, 1991/1992, 1994/1996, 1997/2000.

Co-PI on the NSF SCREMS award to purchase mini-supercomputer hardware, 1993.

Grant-in-aid from the NSF to perform research at the Institute for Advanced Study, Princeton, New Jersey, 1987/88 and 1988/89.

Graduate Division Fellowship (Spencer–Eaken–Almond scholarship) at the University of California, Berkeley, 1983.

Master's degree with distinction 1979.

Polish Academy of Sciences award for an outstanding student 1979.

Dean's Prize awarded at the University of Warsaw, 1974–79.

[1] http://www.ipst.umd.edu/News/awardsandprizes.html

Research Highlights

The solution of the Equichordal Point Problem in 1997 This was one of the most famous problems posed in the language of classical geometry, unsolved since 1916. The problem was described in Encyclopedia Britannica in the "Convex Geometry" section. As with most difficult problems in mathematics, this problem required insights from several diverse fields, including algebraic geometry, dynamical systems, complex analysis and computer explorations. The paper cited below as [20], was published in one of the most prestigious journals in mathematics, and is one of the longest papers devoted to a single mathematical question (significantly longer than the solution of the Fermat's Theorem).

Development of a theory of Lorenz attractors 1987–1991 These attractors were discovered by Edward Lorenz in the 1960's in his study of atmospheric phenomena, and many agree that the discovery began the theory of "Chaos". For my contribution to the theory I received the Monroe Martin Prize. The paper cited below as [10].

Development of a software package *CGBLisp* 1994–present The software analyzes polynomial equations with parameters using the notion of a Comprehensive Gröbner Basis (CGB). My software was used for several years in an independent commercial package called RACER[2]. The authors of the system describe it as follows: "The RACER system is a knowledge representation system that implements a highly optimized tableau calculus for a very expressive description logic." Parts of *CGBLisp* are distributed with Maxima[3], an open source Computer Algebra System originally sponsored by the US Department of Energy.

Research in Mathematical Economics 1999–present I performed research on *labor-managed oligopolies*, a kind of competition model between a number of firms making the same product. The model is applicable to socialist-type economies as well as new types of economies (ESOP schemes, J-firms and codetermined firms). The main assumption of the model is that the surplus (profit) of every individual firm is divided evenly amongst the workers and that the price is inversely proportional to the total production (hyperbolic price assumption). Jointly with my former student, Weiye Li, we were able to prove the existence of Nash equilibria for this model. We also showed that under natural assumption the production in this model must grow like a cubic root of time, while the market share of an individual firm becomes fixed. The research is contained in a series of papers [2, 29, 28].

Software Projects

Lossless Image, Video and Audio Compression

I have served as a Senior Research Mathematician and Chief Science Officer at QBit, LLC, Bethesda, MD, I have worked on cutting edge, proprietary technologies for image, video and data compression.

Problem formulation Digital images are obtained by digitizing data from a variety of analog devices: digital cameras, scanners and medical equipment (CAT scans, X-ray images, MRI, etc). They can be also generated by computers. A full-featured movie like *Madagascar* consists of thousands of images,

[2] http://www.racer-systems.com
[3] http://maxima.sourceforge.net

each occupying about 8MB of storage. The shear amount of data to be stored and moved across communication channels is staggering, dictated by new applications: High Definition Television, Digital Cinema, lossless storage of medical records, deep space sattelite imaging, military applications, etc.

Two problems arise: the cost of storage and the speed of transmission of vast amounts of data associated with images and video. Therefore, there is a considerable interest in development of technologies for lossless compression of data, i.e. in replacing the original data with equivalent data which occupy much less storage, and from which the original data can be efficiently recreated.

Algorithms Data compression revolves around mathematical algorithms which perform data transformations to the compressed form. Many of image and data compression algorithms have become open standards, available for anyone to study: JPEG, JPEG 2000, MPEG2 and MPEG4, JPEG LS, PNG, etc. A piece of software handling compression is divided into two parts, compressor and decompressor. Together, they are called a "codec". A typical compressor is a predictor-corrector system. A well known prediction technique for image compression is the three-point prediction, which predicts the next pixel value from the previous three pixel values, presented in "scan order". The difference between predicted and corrected value is subject to *source coding*, examples of which include *RLE (run-length encoding)*, *Huffman coding*, *arithmetic coding*.

Hardware implementation I formed and have been leading a team of hardware engineers to implement the first FPGA-based hardware design of Qbit. This project, currently near completion will result in a high-performance PCI-X and PCI-E family of FPGA-based boards for high-end video processing markets. These boards will have an unprecedented throughput of 300 MB/s (megabytes per second!) which is produced by moder motion picture industry cameras (2k content, resolution of 2048 by 1556 pixels, or 1080p with resolution of 1920 by 1080 pixels, with color depth of 10 bits per color sample). The boards are equipped with powerful Virtex 5 FPGA chips from Xilinx, implementing the Qbit lossless compression algorithm. I have developed the 64-bit variant of arithmetic coding used in these boards and worked closely with the VP of hardware engineering on its hardware implementation.

A Computer Algebra System for analyzing systems of polynomial equations with parameters implemented in Common Lisp

Problem formulation Many math, science and engineering problems lead to systems of polynomial equations like this one (coming from chemistry): $x_4 - a_4 + a_2 = 0$, $x_1 + x_2 + x_3 + x_4 + a_1 + a_3 + a_4 = 0$, $x_1x_3 + x_1x_4 + x_2x_3 + x_3x_4 - a_1a_4 - a_1a_3 - a_3a_4 = 0$, $x_1x_3x_4 - a_1a_3a_4 = 0$. We want to determine how many solutions in variables x_1, x_2, x_3, x_4 exist, depending on the values of the parameters a_1, a_2, a_3, a_4.

Algorithm This question can be answered algorithmically using the notion of a Comprehensive Gröbner Basis (CGB) developed in 1992 by Weispfenning. This is a complex algorithm and requires building up a number of algorithms for processing systems of polynomial equations.

Software I developed *CGBLisp*, a sizable computer algebra system written in Common Lisp. It performs symbolic processing of systems of polynomial equations with parameters. Amongst its unique features is an interface for automatic geometric theorem proving. This software is available for download[4] at my Web site. Currently, a vastly improved second version is being prepared for release.

[4]http://alamos.math.arizona.edu/ rychlik/symcomp.html

Related projects Along with Common Lisp implementation, I wrote parts of the system for comparison in Prolog, Java and C++. My former graduate student, W. Dunn III, implemented the CGB algorithm in *Macsyma*.

Paper *CGBLisp* is described in a refereed journal article [24].

The RACER[5] project *Racer* is a commercial software package for implementing *Semantic Web* technologies (cf. Racer home page[6]). *Racer* adapted *CGBLisp* as one of the components and used it in its software for a number of years. Racer Systems claims that it does not use my software anymore.

A C++ class template library and an object-oriented environment for simulating large colliding particle systems with a Tcl/Tk graphical user interface

Problem formulation When a large number of balls is confined to a rectangular box and is undergoing elastic collisions with each other and with the walls of the container, various macroscopic quantities, like pressure and temperature, can be measured experimentally. However, the theory which connects the microscopic quantities like momentum and energy to the macroscopic quantities has not yet been fully developed.

Algorithm *DynSys* implements the laws of classical mechanics augmented with collision rules, in an object-oriented fashion. This allows one to easily play with various elastic and non-elastic collision rules, and quickly obtain simulators for new systems. The innovative approach allows extremely accurate measurements of microscopic quantities, decay of correlations and macroscopic limits.

Software *DynSys* is a collection of C++ class templates which allows one to build a simulator of a large particle system in an object-oriented fashion. Most of the classes in *DynSys* are built on the foundation provided by the Standard Template Library (STL). When I started the project in 1997, STL had just become a part of the C++ language, and the development of DynSys allowed me to track the development of STL into a powerful library of reusable objects that it is today. In addition, *DynSys* has a particle simulator with a GUI developed with Tcl/Tk.

Related projects I developed a multi-threaded version of the package using plain C and GTK graphical toolkit, which takes advantage of the parallel hardware in order to handle larger systems.

A Java applet featuring a custom programming language for numerical solution of differential equations

Problem Formulation A modern introductory course in Differential Equations must contain a numerical component which would allow students to conduct numerical experiments. Big software packages like *Mathematica* or *MATLAB* are too complicated and expensive at this level, and have a learning curve steep enough not to be useful in instruction. Therefore, many instructors would give up on the idea of giving students hands-on experience with numerical algorithms.

Software *JOde*[7] is a Java applet which provides easy access to numerical solution of systems of differential equations. The applet packs 12,000 lines of Java code in a 130kb download. This makes the program accessible even for use over slow phone lines. Since no software installation is involved, even

[6] http://www.racer-systems.com
[7] http://alamos.math.arizona.edu/ODEApplet/index.html

students who only use publicly available computer facilities can use the program with virtually any Web browser. Despite the tiny footprint, *JOde* includes a byte compiler for a custom programming language constructed with JavaCC (a Java compiler-compiler). That is why it is likely to be the fastest Web-based differential equations solver. The applet uses the certificate mechanism to allow printing and Java Serialization API to provide persistent storage of client data.

Popularity *JOde* is used by many universities and individuals around the world. For instance, at the University of Arizona the program is used for homework assignments by all students taking introductory differential equations. Although designed for educational use, *JOde* has been used in research as well.

A thread pool library for concurrent and parallel computation in C and C++

An abstract thread pool toolkit On top of the POSIX Threads library, I wrote a C library which implements the thread pool paradigm of concurrent computing, with only three top-level functions. The library is aimed at fast conversion of existing single-thread applications to a multi-threaded, multiprocessor environment. The library has already been used to convert two programs to run on a parallel supercomputer, and it showed excellent performance characteristics (95% processor utilization with 64 processors for our test application).

Billiard systems in a triangle The thread pool library was used to parallelize the C code and C++ code related to a famous mathematical question concerning the existence of obtuse triangles with no periodic billiard trajectories.

Concurrent Gröbner basis calculation In collaboration with a graduate student, I am working on a high performance code for calculating Gröbner bases, crucial in the theory of systems of polynomial equations.

Mathematica, Maxima and Maple

Mathematica I developed a substantial collection of packages and notebooks[8] in *Mathematica*, the well-known Computer Algebra System from Wolfram Research. Some of the notebooks are available from my Website. They cover such applications as computing fractal dimension, proof of the existence of the Feigenbaum fixed point, computation of the curvature tensor in differential geometry, calculating natural measures for dynamical systems and others.

MACSYMA and Maxima I have written large amounts of code for the MACSYMA/Maxima Computer Algebra Systems. I contributed a Gröbner basis package to the *Maxima*[9] *project. The package is distributed with the newest version of* Maxima*. The package is written in Common Lisp, just as* Maxima *itself. I am intimately familiar with the details of the implementation of the* Maxima *system.*

Maple *I have written Maple codes for automatic geometric theorem proving.*

[8]http://alamos.math.arizona.edu/ rychlik/notebooks.html
[9]http://maxima.sourceforge.net

A C program with an XView GUI for simulation and analysis dynamical systems with planar, cylindrical and toral phase spaces

Problem formulation Many important physical systems can be modeled with mappings of a plane, a cylinder or a torus: a periodically forced pendulum, predator-prey models, Taylor-Greene-Chirikov map, Hénon-Heiles equation, mappings defined by a rational function in the complex plane and others. Many fundamental phenomena like chaos, invariant curves and periodic solutions are only accessible by a numerical experiment. Thus a need for a scientific visualization environment.

Algorithms Iteration of points and curves of explicit and implicit systems, calculation of Julia sets, calculations of equilibria and invariant manifolds, a sophisticated zoom feature, automatic code generation and other numerical methods.

Software I implemented the original version of *Iterator* in 1987. Since then *Iterator* has been updated many times. Currently, *Iterator* is written in C and XView, a GUI toolkit from SUN. *Iterator* is available for Unix platforms. Since 2002 the package uses the GNU automake to make the package autoconfigurable. The package supports automatic code generation and two dynamic loading schemes (one based on *dlfcn.h* and another based on *dld*) which allow the package to run on several versions of Linux and other UNIX flavors. *Iterator* is now distributed using RedHat's RPM packaging tool. It is available for download at my Web site [13].

A C/FORTRAN program with a Tcl/Tk GUI simulating a certain important Hamiltonian system with elastic collisions

Problem formulation A mechanical system of balls is studied. Balls undergo collisions with a common floor and interact with the floor according to a linear potential. Equipartition of energy, decay of correlations and other statistical properties of trajectories are investigated in the context of the theory of chaotic dynamical systems.

Algorithms Equations of motion solved directly. Ergodic averages of energy calculated using Newton-Cotes formulas of integration. A new method for computing the correlation functions is used to measure the time decay of correlations extremely accurately. A non-linear optimization package written in FORTRAN called ODRPACK is used to solve an interpolation problem for the correlation function.

Software The simulation software is developed in C and FORTRAN. The program has a GUI based on the Tcl/Tk toolkit with the BLT widget library. The Tcl/Tk layer communicates with the C and FORTRAN programs using pipes. Real-time simulation of > 1000 balls can be run easily. Program is packaged using GNU autoconfiguration tools and distributed from my Web site.

Computer Graphics, Virtual Reality and Ray Tracing

An Object-Oriented Ray Tracing Program I designed and implemented in C++ an object-oriented ray tracing program, which I used in a graduate course in Computer Graphics I taught as an illustration of object oriented design and realistic rendering. The program is available at my website.

Virtual Reality I studied and taught Virtual Reality techniques using VRML and Java3D. I developed a collection of Virtual Reality models for Calculus instruction.

A PHIGS-like Library I designed and implemented under X Windows a C++ graphics library maintaining display lists of graphical objects.

Artificial Intelligence, Logic Programming and Automated Reasoning

Lambda calculus I implemented an interpreter of untyped lambda calculus in Common Lisp and Scheme. I implemented a typed lambda calculus based on the Hindley-Milner theorem, using a unification algorithm in Common Lisp. I am developing a larger system aimed at discovering and proving new mathematical theorems, featuring first-order logic with equality. The prototype of the system uses unification and paramodulation, and I am working on efficient processing of axiom schema. With this feature, the system will allow encoding of most axiomatic theories of mathematics.

Automated Reasoning I wrote resolution-based inference engines for automatic theorem proving in Prolog and in Common Lisp. I used these programs as part of the curriculum of a graduate course which I taught.

General Problem Solver I designed and implemented a version of General Problem Solver (GPS), blocks world solver (Common Lisp) and a Prolog meta-interpreter with probabilistic logic (in Prolog).

Scientific Meeting Participation and Organization

- Invited talk at the sectional meeting of the AMS, Los Angeles, California, April 2004.
- Invited talk at the national meeting of the AMS, Phoenix, Arizona, January 2004.
- Co-organizer and speaker at the special session of the Sectional AMS meeting, Irvine, CA, November 2001.
- Invited Speaker at the Dynamical Systems seminar, Austin, Texas, April 2001.
- Invited speaker at the Southwestern Dynamical Systems Workshop, Los Angeles, November 2000.
- Invited speaker at the Sectional AMS meeting, Birmingham, Alabama, November 2000.
- Invited speaker at the Sectional AMS meeting, Santa Barbara, California, March 2000.
- Invited speaker at the Sectional AMS meeting, Tucson, Arizona, November 1998. Co-organizer of a special session on Dynamical Systems.
- Invited address at the Sectional AMS meeting, Atlanta, Georgia, October 1997. Organizer of a special session related to the address.
- Invited speaker at the Sectional AMS Meeting, Corvallis, Oregon, April 1997.
- Invited address at the Dynamics Days, Tempe, Arizona, January 1997.
- Invited speaker at the Sectional AMS Meeting, Pasadena, CA, November 1996.
- Invited speaker at the Rutgers meeting on Hyperbolic Dynamics and Applications to Non-equilibrium Statistical Mechanics, Rutgers, New Brunswick, October 1996.

- Invited speaker at the 1st Southwest Dynamical Systems Workshop, Tucson, March 1996.
- Invited speaker Banach Center Symposium, Ergodic Theory and Dynamical Systems, Warsaw, Poland, June 1995. Delivered four lectures on current research. Co-organizer of two special sessions.
- Invited speaker at the Third SIAM Conference on Applications of Dynamical Systems, Snowbird, Utah, May 1995. Organizer of a mini-symposium.
- Invited speaker at the Dynamical Systems and Related Topics Workshop, Penn State, October 13-16, 1994.
- Invited speaker at the 13th Annual Western States Mathematical Physics Meeting, California Institute of Technology, February 28 and March 1, 1994.
- Invited speaker at the International Conference/Workshop on Dynamical Systems, IMPA, August 2-15, 1993.
- Invited speaker at the Midwest Dynamical Systems Seminar, Boulder, Colorado, March 25-28, 1993.
- Invited speaker at the Penn State/University of Maryland Workshop, College Park, March 11-13, 1993.
- Participant of the four conferences organized by the Regional Dynamics Institute, Boston University, June 29—July 25, 1991.
- Delivered the Monroe Martin Prize lecture, University of Maryland, College Park, 1991. Participated in a joint University of Maryland—Penn State meeting on Dynamical Systems and Ergodic Theory.
- Invited speaker at the Annual SIAM meeting, Chicago, Illinois, 1990.
- Invited speaker at Hoboken, New Jersey, AMS meeting, October 1989.
- Invited speaker at Northwest Dynamical Systems Seminar, Cincinnati, Ohio, 1987.
- Invited speaker at a symbolic computation conference, Cornell University, September 1986.
- Invited speaker at Dynamics Days, La Jolla, California, 1986.
- Invited speaker at a dynamical systems conference, Oberwolfach, Germany, September 1981.

Direction of Dissertation and Thesis Projects

Ph.D. project of Chun-Woo Yang entitled *Parameter Conditions for the Existence of Homoclinic Orbits in the Lorenz Equations*, University of Arizona, Tucson; completed 1994.

Ph.D. project of William Dunn III entitled *Algorithms and Applications of Comprehensive Groebner Bases*, University of Arizona, Tucson; completed 1995.

Ph.D. project of Mark Torgerson devoted to the classification of certain maps admitting polynomial and algebraic integrals; completed 1997.

Masters' thesis project of John Larson addressed the problem of noisy time series prediction using two approaches: neural networks and Takens embedding. The approach was tested on financial data related to the stock market; completed 1999.

Ph.D. dissertation project of Li Weiye entitled *Stability of Equilibria in Dynamic Oligopolies*, University of Arizona, Tucson; completed 2001.

Ph.D. Dissertation project of Andy Linfoot Investigation of symbolic computation techniques in partial differential equations; University of Arizona, Tucson; completed in 2007.

Ph.D. Dissertation Project of Robert Lakatos University of Arizona, Tucson; started in 2003. Robert investigated genetic algorithms, genetic programming and speech recognition.

Professional Service and Outreach

- Elected member of the University of Arizona Committee on Academic Freedom and Tenure, 1998–2001.
- Member of Presidential Committee to evaluate the University of Arizona athletic program, required to gain NCAA certification, University of Arizona, 1997–present.
- Member of Computer Committee, University of Arizona, Department of Mathematics (4 years).
- Member of Graduate Committee, University of Arizona, Department of Mathematics (2 years).
- Member of the Dean's Grade Appeal Committee, University of Arizona.
- Judge and Category Chair at the International Science and Engineering Fair, Tucson, Arizona, May 1996.
- Judge at the Southwest Science Fair, Tucson, Arizona, 1994.
- Participated in grading of the Upper-Division Writing Proficiency Exam for the University Composition Board, University of Arizona, 1994–1997.
- Referee for several mathematical journals.
- NSF and DOE grant reviewer.
- Supervised several undergraduate research projects. Undergraduate research workshop speaker.
- Committee member of approximately 20 oral examinations, Ph. D and Masters.
- Organizer of a special session at the Sectional AMS meeting, Atlanta, Georgia, September 1997.
- Co-organizer of a special session at the upcoming Sectional AMS meeting, Tucson, Arizona, November 1998.
- Mini-symposium organizer at the Third SIAM Conference on Applications of Dynamical Systems, Snowbird, Utah, May 1995.
- Co-organizer of Banach Center Symposium, Ergodic Theory and Dynamical Systems, Warsaw, Poland. Co-organized two mini-symposia (Smooth Ergodic Theory with A. Katok and Low-Dimensional Dynamics with M. Viana), June 1995.

Publications

[1] D. Engler and M. Rychlik. Renormalization of quasiperiodic linear systems and the Shadowing Lemma. Preprint, 1992.

[2] W. Li, M. Rychlik, F. Szidarovszky, and C. Chiarella. On the stability of a class of homogenous dynamic economic systems. *Nonlinear Analysis: Theory, Methods & Applications*, 52(6):1617–1636, 2003.

[3] M. Rychlik. Bounded variation and invariant measures. *Studia Math.*, t. 76:69–80, 1983.

[4] M. Rychlik. *Invariant measures and the variational principle for Lozi mappings*. Ph. D. dissertation, University of California, Berkeley, 1983.

[5] M. Rychlik. Mesures invariantes et principe variational pour d'applications de Lozi. *C. R. Acad. Sci. Paris*, 296, January 1983.

[6] M. Rychlik. About some absolutely continuous invariant measures. Preprint, 1986.

[7] M. Rychlik. Lorenz attractors in unfoldings of some vector fields. Preprint, 1986.

[8] M. Rychlik. Another proof of Jakobson's theorem and related results. *Erg. Theory and Dyn. Sys.*, 8:83–109, 1988.

[9] M. Rychlik. The Wiener lemma and cocycles. *Proc. of the AMS*, 104(3), November 1988.

[10] M. Rychlik. Lorenz attractors through a Šilnikov–type bifurcation, Part I. *Erg. Theory and Dyn. Sys.*, 10:811–840, 1989. Winner of the Monroe Martin Prize, Maryland, 1991.

[11] M. Rychlik. Periodic points of period three of the billiard ball map in a convex domain have measure 0. *J. of Diff. Geometry*, 30:191–205, 1989.

[12] M. Rychlik. Regularity of the metric entropy for expanding maps. *Trans. Amer. Math. Soc.*, 315(2):833–847, October 1989.

[13] M. Rychlik. Iterator. A computer program for visualization of 1- and 2-dimensional dynamical systems. Available for UNIX platforms with XView GUI toolkit from Web site `http://alamos.math.arizona.edu`, 1989–1999.

[14] M. Rychlik. Large deviation estimates for expanding mappings. Preprint, based on a talk given at the Annual SIAM Meeting, Chicago, Illinois, 1990.

[15] M. Rychlik. Renormalization approach to dynamics. Lecture notes based on a graduate course given during Fall 1990 (draft version), 1990.

[16] M. Rychlik. Renormalization of cocycles and linear ODE with almost-periodic coefficients. *Inventiones Math.*, 110:173–206, 1992.

[17] M. Rychlik. About absence of homoclinic connections in the standard map. Preprint, 1993.

[18] M. Rychlik. CgbLisp. A computer algebra system for calculating Comprehensive Gröbner Bases, currently installed at the University of Arizona, Tucson, available from Web site `http://alamos.math.arizona.edu`, 1995–1997.

[19] M. Rychlik. The Equichordal Point Problem. *Electronic Research Announcements of the AMS*, 2(3):108–123, 1996.

[20] M. Rychlik. A Complete Solution to the Equichordal Point Problem of Fujiwara, Blaschke, Rothe and Weizenböck. *Inventiones Math.*, 129(1):141–212, 1997.

[21] M. Rychlik. Mathematics of symbolic computation. HTML document available at Web site `http://alamos.math.arizona.edu`, 1997.

[22] M. Rychlik. Dynsys — Boltzmann Gas Simulator and computational dynamics tools. URL: `http://alamos.math.arizona.edu/DynSys/doxy`, 1997-2003. Class documentation and C++ annotated sources.

[23] M. Rychlik. Complexity and applications of parametric algorithms of computational algebraic geometry. In R. del la Llave, L. Petzold, and J. Lorenz, editors, *Dynamics of Algorithms*, volume 118 of *The IMA Volumes in Mathematics and its Applications*, pages 1–29, New York, 2000. Springer-Verlag.

[24] M. Rychlik. A Gröbner basis package for the computer algebra system *Maxima*. distributed with *Maxima* at the development site `http://maxima.sourceforge.com` and available at the author's website `http://alamos.math.arizona.edu`, May 2002. programmed in *Common Lisp*.

[25] M. Rychlik. Numerical computation of the rate of decay of correlations and other spectral invariants for a system of hard particles. acepted by *Experimental Mathematics* subject to revision, 2002.

[26] M. Rychlik. Another proof of Jakobson's theorem and related results. In B. R. Hunt, J. A. Kennedy, T.-Y. Li, and H. E. Nusse, editors, *The Theory of Chaotic Attractors*, pages 313–334. Springer Verlag, 2003.

[27] M. Rychlik. Invariant measures and the variational principle for Lozi mappings. In B. R. Hunt, J. A. Kennedy, T.-Y. Li, and H. E. Nusse, editors, *The Theory of Chaotic Attractors*, pages 190–221. Springer Verlag, 2003.

[28] M. Rychlik and W. Li. Dynamic equilibria in homogenous systems and a labor-managed oligopoly model. Preprint, 2003.

[29] M. Rychlik and W. Li. Global attractivity of Nash equilibria of a labor-managed oligopoly model. Submitted, 2003.

[30] M. Rychlik and E. Sorets. Regularity and other properties of absolutely continuous invariant measures for the quadratic family. *Comm. Math. Phys.*, 150:217–236, 1992.

[31] M. Rychlik and M. Torgerson. Algebraic non-integrability of the Cohen map. *New York Journal of Mathematics*, 4:57–74, October 1998.

[32] M. Rychlik and C.-W. Yang. Homoclinic connections in the Lorenz system. Preprint, 1994.