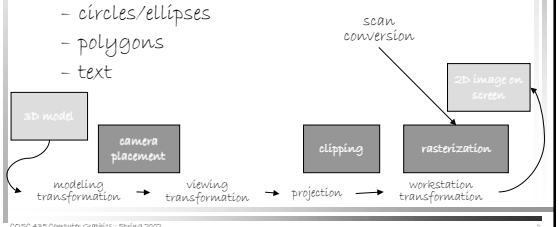
Scan-Conversion

Scan-Conversion

- figuring out what pixels have to be colored in order to draw a geometric primitive on the screen
  - lines
  - circles/ellipses
  - polygons
  - text



Scan-Conversion Overview

- lines
  - DDA
  - midpoint/Bresenham
- circles
  - midpoint circle
- filled polygons
  - scan-line
- antialiasing

Scan-Conversion: Lines

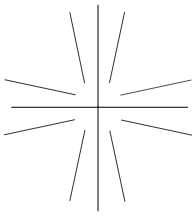
the task: draw a line on a raster screen between two points

- mathematical line vs. rasterized line
  - jaggies

revised task: given two integer-coordinate points on the plane, determine what pixels on a raster screen should be on to create a picture of a unit-width line segment between those points

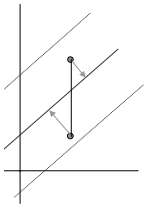
A Simplification

- we'll consider only lines with slope $0<m<1$
  - horizontal and vertical lines are special cases
  - diagonal lines ($m = \pm 1$) can be special-cased or included in general case
  - can use symmetry to adapt solution for $0<m<1$ for $m > 1$ and negative slopes



Another Simplification

- consider line with slope $0<m<1$
  - generally only one pixel per column inside the unit-width line segment
  - we'll draw just one pixel per column, using the closest to the "true line" if there are two choices
  - vertical distance from point to line is proportional to perpendicular distance



Lines: Basic Algorithm

- find the equation of the line connecting endpoints P & Q
- starting with the leftmost point P, $x_i = x_{i-1}+1$ and $y_i = m \cdot x_i + b$
- color pixel at $(x_i, round(y_i))$
- each iteration requires floating-point multiplication

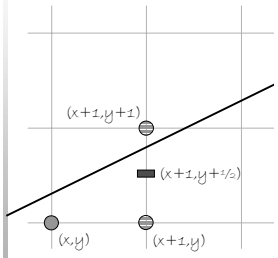
Lines: DDA Algorithm

- determine slope of the line connecting endpoints P & Q
- starting with the leftmost point P, $x_i = x_{i-1}+1$ and $y_i = y_{i-1}+m$
- color pixel at $(x_i, round(y_i))$
- each iteration still requires floating-point arithmetic and rounding
- repeated summing of fractional values can lead to roundoff problems for very long lines

Lines: Midpoint Line Algorithm

- observation: the next pixel colored is always either E or NE of the current pixel
- need a way to decide between the choices…
  - (and with only using integer arithmetic)
- let's consider the midpoint between the two choices…

The Midpoint



- if the line goes between the midpoint and the NE point, NE is closer
- if the line goes between the midpoint and the E point, E point is closer

Equation of a Line

- one way to express a line
  $y = mx+B = (\Delta y/\Delta x)x+B$
  - m is the slope, B is the y intercept
- another way of expressing a line
  $F(x,y) = ax+by+c = 0$
  rearranging the first equation yields
  $(\Delta y)x - (\Delta x)y + (\Delta x)B = 0$
  - $F(x,y) = 0$ if (x,y) is on the line
  - $F(x,y) < 0$ if (x,y) is above the line
  - $F(x,y) > 0$ if (x,y) is below the line

Putting the Bits Together

- if midpoint is below the line, then line is between midpoint and NE
  - if $F(x_m, y_m) > 0$, choose NE
- if midpoint is above the line, then line is between midpoint and E
  - if $F(x_m, y_m) < 0$, choose E
- if midpoint is on the line, then line goes through the midpoint
  - if $F(x_m, y_m) = 0$, choose either E or NE (but be consistent)
- we'll write $F(x_m, y_m)$ as d and call it the decision variable

Notation Roundup

- line endpoints are $P=(x_p,y_p)$ and $Q=(x_q,y_q)$
  - $\Delta x = x_q - x_p$, $\Delta y = y_q - y_p$
  - P and Q have integer coordinates
- pixels are colored one column at a time
  - (x_k,y_k) are the coordinates of the pixel colored in column k, $0 \le k \le \Delta y$ (k is an integer)
    - $x_{k+1} = x_k + 1$
  - $(x_0,y_0) = P$, $(x_{\Delta y},y_{\Delta y}) = Q$
  - $d_k = F(x_{k-1}+1, y_{k-1}+1/2)$
    - d_k is decision variable used to choose where pixel in column k goes

An Algorithm Based on the Midpoint

- to compute (x_{k+1},y_{k+1})...
  - midpoint is $(x_k+1, y_k+1/2)$
  - compute $d_{k+1} = F(x_k+1, y_k+1/2)$
  - $x_{k+1} = x_k + 1$
  - if $d_{k+1} > 0$, $y_{k+1} = y_k + 1$
  - otherwise, $y_{k+1} = y_k$
- this works, but computing $F(x_k+1, y_k+1/2)$ involves several floating-point additions & multiplications
  - we're trying to eliminate floating point!

Making It Incremental

- if $d_{k+1} = d_k + \Delta$ we could save work

$$\begin{aligned} d_{k+1} - d_k &= F(x_k+1, y_k+1/2) - F(x_{k-1}+1, y_{k-1}+1/2) \\ &= (\Delta y)(x_k+1) - (\Delta x)(y_k+1/2) + (\Delta x)B - \\ &\quad [(\Delta y)(x_{k-1}+1) - (\Delta x)(y_{k-1}+1/2) \\ &\quad + (\Delta x)B] \\ x_k = x_{k-1}+1 \quad &= (\Delta y)(x_k - x_{k-1}) - (\Delta x)(y_k - y_{k-1}) \\ &= (\Delta y)(x_{k-1}+1-x_{k-1}) - (\Delta x)(y_k - y_{k-1}) \\ &= \Delta y - (\Delta x)(y_k - y_{k-1}) = \Delta \end{aligned}$$

- relationship between y_k and y_{k-1} depends on whether we chose NE or E pixel for (x_k,y_k)

Making It Incremental

$$d_{k+1} - d_k = \Delta y - (\Delta x)(y_k - y_{k-1})$$

- relationship between y_k and y_{k-1} depends on whether we chose NE or E pixel for (x_k,y_k)
- if E... $y_k = y_{k-1}$

  $d_{k+1} - d_k = \Delta y = \Delta_E$
- if NE... $y_k = y_{k-1} + 1$

$$\begin{aligned} d_{k+1} - d_k &= \Delta y - (\Delta x)(y_{k-1}+1-y_{k-1}) \\ &= \Delta y - \Delta x = \Delta_{NE} \end{aligned}$$

- Δ_E and Δ_{NE} are constants...

The Incremental Approach

- to compute (x_{k+1},y_{k+1})...
  - assume d_{k+1} has already been computed
  - $x_{k+1} = x_k + 1$
  - if $d_{k+1} > 0$, $y_{k+1} = y_k + 1$ and $d_{k+2} = d_{k+1} + \Delta_{NE}$
  - otherwise, $y_{k+1} = y_k$ and $d_{k+2} = d_{k+1} + \Delta_E$

Initial Values

- but what are x_0, y_0, d_1?
  - x_0 and y_0 are the coordinates of the lower left endpoint

$$\begin{aligned} d_1 &= F(x_0+1, y_0+1/2) \\ &= (\Delta y)(x_0+1) - (\Delta x)(y_0+1/2) + (\Delta x)B \\ &= (\Delta y)x_0 - (\Delta x)y_0 + (\Delta x)B + \Delta y - \Delta x/2 \\ &= F(x_0,y_0) + \Delta y - \Delta x/2 \end{aligned}$$

  - (x_0,y_0) is an endpoint and thus on the line, so $F(x_0,y_0) = 0$

    $d_1 = \Delta y - \Delta x/2$
  - but $\Delta x/2$ may not be integer!

Making It Integer

$d_1 = \Delta y - \Delta x/2$

- d_1 may not be integer
  - so, multiply by 2! (Δy and Δx are integers)
  - this doesn't affect the sign of d, which is all that matters
- we now use $d_{k+1} = 2 \cdot F(x_k+1, y_k+1/2)$
  - so $d_1 = 2\Delta y - \Delta x$
  - this also means that Δ_E and Δ_{NE} are twice as big: $\Delta_E = 2\Delta y$ and $\Delta_{NE} = 2\Delta y - 2\Delta x$

$0 < m < 1$ Midpoint Line Algorithm

- compute $2\Delta x$, $2\Delta y$, and $2\Delta y - 2\Delta x$
- initialize (x,y) to the lower leftmost point and $d = 2\Delta y - \Delta x$
- color pixel (x,y)
- repeat...
  - increment x
  - if $d > 0$, increment y and add $\Delta_{NE} = 2\Delta y - 2\Delta x$ to d
  - otherwise, add $\Delta_E = 2\Delta y$ to d
  - color pixel (x,y)

Midpoint Line Summary

- algorithm is integer & incremental (for speed)
- simplifications
  - one pixel per column (a hack to make it easier)
  - only consider $0 < m < 1$ (other cases by symmetry)
- key observation: given pixel colored in column k, next pixel is either E or NE
  - need a decision variable to use to make choice

Midpoint Line Summary #2

- decision variable
  - use sign (+, -, 0) to make choice
  - the sign of F(x,y) tells which side of the line point (x,y) is on
  - determine what side of line midpoint is on
    - if line is between midpoint and E, E is closer to line i.e. choose E if $F(x_m, y_m) < 0$
    - ditto for NE i.e. choose NE if $F(x_m, y_m) > 0$
- making it integer & incremental
  - use 2F(x,y) as decision var instead of F(x,y)
  - don't recompute F(x,y) each time – instead, increment previous value by some amount